1. SOME TERMS RELATING TO CIRCLES

1.1 DEFINITION OF A CIRCLE

A circle is a closed figure in a plane and it is the collection of all those points in the plane, which are at a constant distance from a fixed point in the plane. The fixed point is called the **centre** of the circle and the constant distance is called the **radius** of the circle.

1.2 CIRCUMFERENCE OF A CIRCLE

The circumference of a circle is the length of the complete circular curve constituting the circle.

1.3 CHORD OF A CIRCLE

In figure line segment *AB* joining two points *A* and *B* of the circle is called a **chord** of the circle.

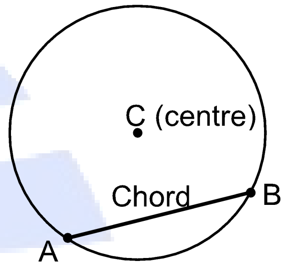
A chord passing through the centre *C* of the circle is called a **diameter** of the circle. A diameter of a circle is the longest chord of the circle and its length is twice the radius of the circle.

We have

Diameter of a circle = 2 × radius of the circle

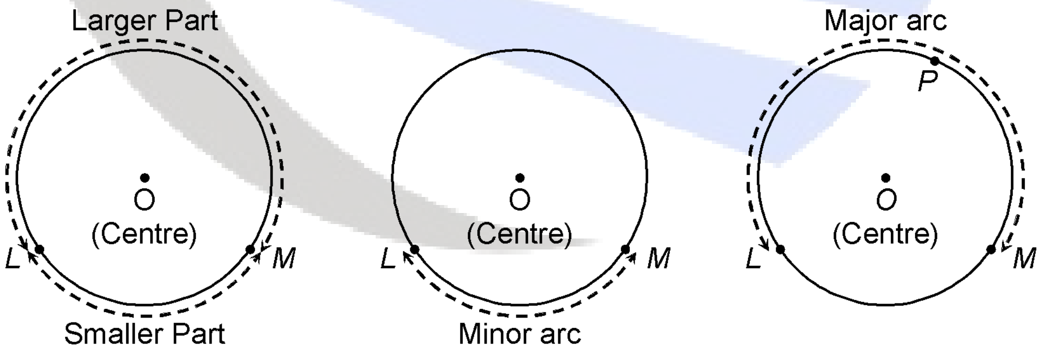
i.e., $D = 2r$

Here, *D* is the length of diameter and *r* is the radius of the circle.



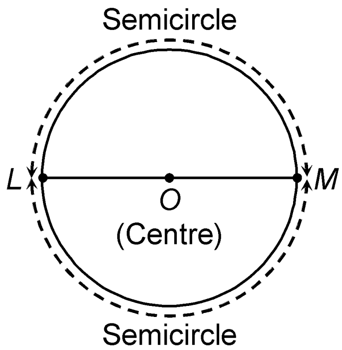
1.4 ARC OF A CIRCLE

Any two points *A* and *B* of a circle, divide the circle in two parts as shown in figure (i). The smaller part is called a minor arc of the circle as shown in figure (ii), and it is denoted by $\overset{\frown}{LM}$ (read as arc *LM*). The larger part is called a major arc as shown in figure (iii). It is denoted by $\overset{\frown}{LOM}$ where *P* is a point on this part of the arc. We read it as are *APB* or as major arc *LM*.



(i) **(ii)** **(iii)**

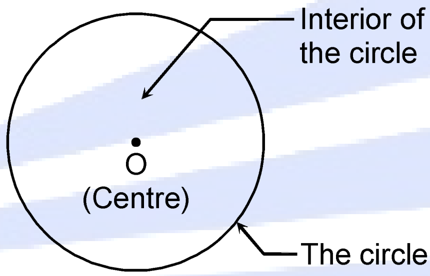
In case the two parts are equal, then we find *LM* is a diameter of the circle and two parts of the circle are equal. Each part of the circle as shown in figure (iv) is a **semicircle.**



(iv)

1.5 INTERIOR OF A CIRCLE

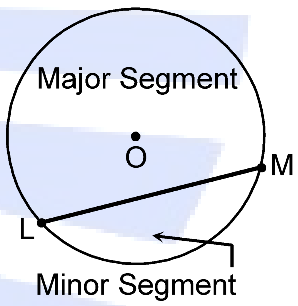
The plane region lying the circle as shown in figure is called the **interior** of the circle.



The circle and its interior both as a whole constitutes the **circular region.**

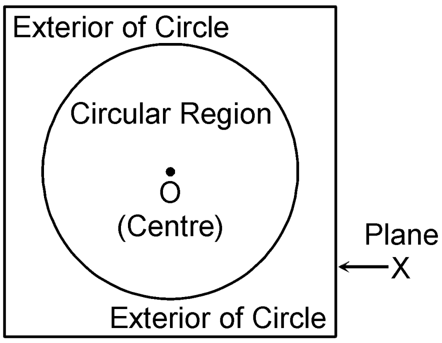
1.6 SEGMENT OF A CIRCULAR REGION

A chord *LM* divides a circular region in two parts as shown in figure. The smaller part is called the **minor segment** and the larger part is called the **major segment** of the circular region. In case chord *LM* is a diameter of the circle, then the two segments are equal and each is called as **semicircular region.**



1.7 EXTERIOR OF A CIRCLE

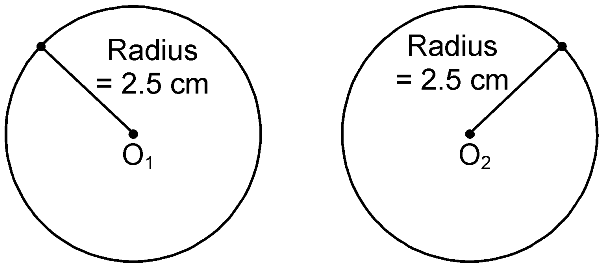
If a circle be draw in the plane *X* (infinite dimensions), then the part of the plane region outside the circular region is called the exterior of the circle as shown in figure.



1.8 CONGRUENT CIRCLES

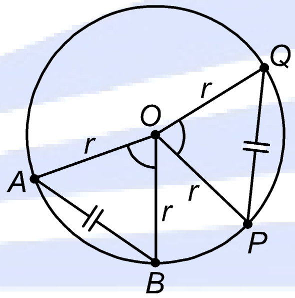
Two circles are congruent to each other if and only if they have equal radii.

In the figure two circles have equal radii. Therefore, the two circles are congruent to each other.



Theorem 1 : Equal chords of a circle subtend equal angles at the centre of the circle.

Given : A circle having centre at O and radius r is given. AB and PQ are its equal chords. $\angle ACB$ and $\angle PCQ$ are the angles subtended by the chords AB and PQ respectively at the centre O.

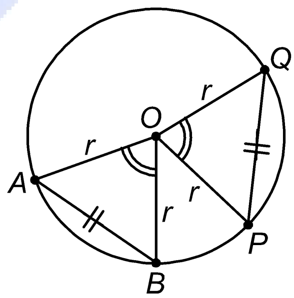


To prove : $\angle AOB = \angle POQ$.

Proof : In ΔOAB and ΔOPQ, we have

	$OA = OP$	[Each = radius r]
	$OB = OQ$	[Each = radius r]
	$AB = PQ$	[Given]
Therefore,	$\Delta OAB \cong \Delta OPQ$	[By SSS congruence criteria]
We have	$\angle AOB = \angle POQ$.	[cpct]

Theorem 2 : If the angles subtended by the chords of a circle at the centre are equal, then the chords are equal.



Given : A circle having centre at O and radius r is given. $\angle AOB$ and $\angle POQ$ are subtended by chords AB and PQ respectively at the centre O such that $\angle AOB = \angle POQ$

To prove : Chord AB = Chord PQ

Proof : In ΔOAB and ΔOPQ, we have

	$OA = OP$	[Each = radius r]
	$\angle ACB = \angle POQ$	[Given]
	$OB = OQ$	[Each = radius r]
Therefore,	$\Delta OAB \cong \Delta OPQ$	[By SAS congruence criteria]

Then by cpct, we have $AB = PQ$.

2. PREPENDICULAR FROM THE CENTRE TO A CHORD

Theorem 3 : The perpendicular drawn from the centre of a circle to a chord of the circle bisects the chord.

Proof : In ΔOAM and ΔOBM

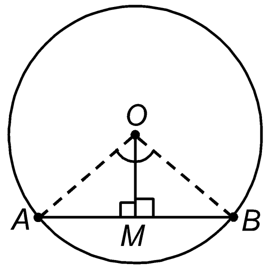
$OA = BO$ [Each = radius]

$OM = OM$ [Common side]

and $\angle OMA = \angle OMB$ [both 90°]

$\Delta OAM \cong \Delta OBM$

[By RHS congruence criteria)



Then by cpct, we have $AM = BM$. Thus, OM bisects the chord AB.

Theorem 4 : The line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

Proof: Line ⟊ passes through the centre O of the circle and bisects the chord AB at M. i.e.,

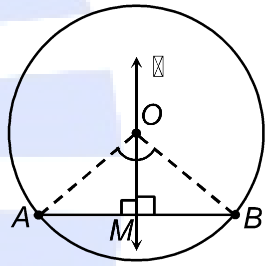
$AM = BM$

In ΔOAM and ΔOBM, we have

$OA = OB$ [Each = radius]

$OM = OM$ [Common side]

$AM = MB$ [Given]



Therefore, $\Delta OAM \cong \Delta OBM$ [By SSS congruence]

$\angle OMA = \angle OMB$ [cpct]

Also, $\angle OMA + \angle OMB = 180^\circ$ [A linear pair of angles]

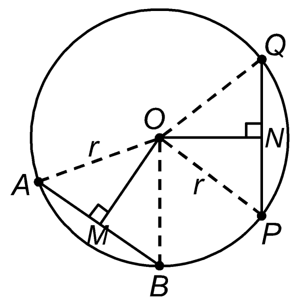
$\Rightarrow$ $\angle OMA = \angle OMB = 90^\circ$

Thus, we have $OM \perp AB$ i.e., $l \perp AB$.

3. EQUAL CHORDS AND THEIR DISTANCES FROM THE CENTRE OF A CIRCLE

Theorem 5 : Equal chords of circle (or of congruent circles) are equidistant from the centre (or centres of respective circles).

In the figure, O is the centre of the circle and r is its radius. Chord AB = Chord PQ·$OM \perp AB$ and $ON \perp PQ$. Now, we have



$AM = BM = \frac{1}{2}AB$

and $PN = QN = \frac{1}{2}PQ$

⇒ $AM = BM = PN = QN$ [∵ $AB = PQ$]

In ΔOAM and ΔOPN, we have

$OA = OP$ [Each = radius]

$AM = PN$ [Proved]

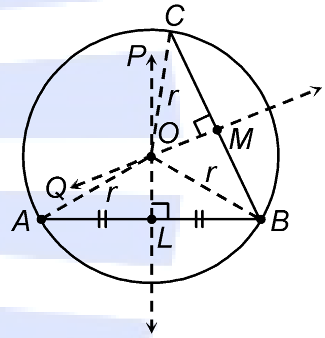
and $\angle OMA = \angle ONP$ [Each = 90º]

Therefore, $\Delta OAM \cong \Delta OPN$ [RHS congruence criteria]

By cpct, we have $OM = ON$ i.e., AB and PQ are equidistant from the centre O.

Theorem 6 : (Converse of Theorem 5). Chords equidistant from the centre of a circle (or from the respective centres of congruent circles) are equal in length.

4. CIRCLE THROUGH THREE POINTS



Theorem 7 : There is one and only one circle passing through three given points.

Given : Three non-collinear coplanar points A, B and C.

To prove : There is one and only one circle passing through the points A, B and C.

Construction : Draw line-segments AB and BC. Construct ⊥ bisectors PL and QM of AB and BC respectively and they intersect at O (say). Join OA, OB and OC.

Proof : In ΔOAL and ΔOBL $OL = OL$ [Common side]

$AL = BL$ [By Construction]

$\angle OLA = \angle OLB$ [Both 90º]

⇒ $\Delta OAL \cong \Delta OBL$ [By RHS congruence]

⇒ $OA = OB$ [cpct]

Similarly, $OB = OC$ [cpct]

Thus, $OA = OB = OC = r$

Circle of radius r, with centre at O will pass through the points A, B and C.

Uniqueness of circle passing through A, B and C:

Let if possible, there be another circle passing through the points A, B and C. Let its centre be at O' and radius r'.

Now O' must lie on PL as well as on QM.

i.e., O' is intersection point of PL and QM.

⇒ $O'A = OA = r$

⇒ $O'A = O'B = O'C = r$

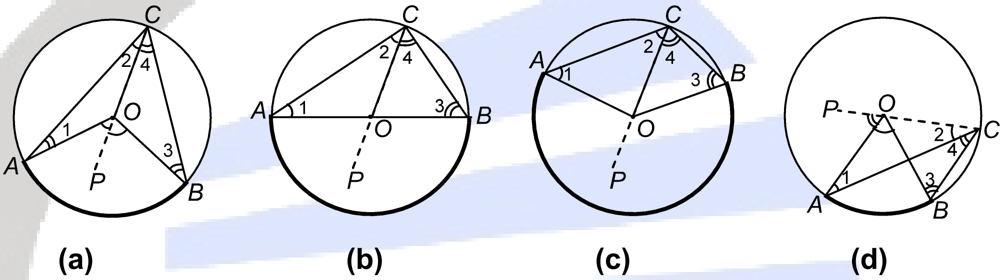
Hence, there is one and only one circle passing through the points A, B and C.

5. ANGLE SUBTENDED BY AN ARC OF A CIRCLE

Theorem 8 : The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

Given : A circle with centre O and $\overset{\frown}{AB}$ of the circle. $\overset{\frown}{AB}$ subtends $\angle AOB$ at the centre and $\angle ACB$ at any point C on the remaining part of the circle.

In figure (a) and (d) $\overset{\frown}{AB}$ is minor, in figure (b) $\overset{\frown}{AB}$ is a semicircle and in figure (c) $\overset{\frown}{AB}$ is major arc.



To prove : $\angle AOB = 2\angle ACB$ [Figure (a), (b) and (d)]

and reflex $\angle AOB = 2\angle ACB$ [Figure (c)]

Construction : Join CO and produce CO to P.
Join OA and OB

Proof : In ΔOAC, we have

$OA = OC$ [radii of same circle]

$\Rightarrow \angle 1 = \angle 2$ [Angles opposite to equal sides] …(i)

Now, $\angle AOP = \angle 1 + \angle 2 = 2\angle 2 = 2\ \angle ACO$ [Exterior angle property]

$\therefore \angle AOP = 2\angle ACO$ …(ii)

Similarly, $\angle BOP = 2 \times \angle 4 = 2\angle BCO$

$\therefore \angle BOP = 2\ \angle BCO$ …(iii)

For figure (a), Adding (ii) and (iii), we have

$\angle AOP + \angle BOP = 2$ $[\angle ACO + \angle BCO]$

$\therefore \angle AOB = 2\angle ACB$

For figure (b), we have

$\angle AOB = 2\angle ACB$

For figure (c)

$\angle AOP + \angle BOP = 2(\angle ACO + \angle BCO)$

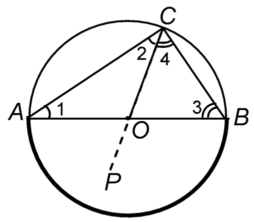
$\Rightarrow$ reflex $\angle AOB = 2\angle ACB$

For figure (d), Subtracting (ii) and (iii), we have

$\angle BOP - \angle AOP = 2(\angle BCO - \angle ACO)$

It gives, $\angle AOB = 2\angle ACB$

Corollary : The angle subtended in a semicircle is a right angle.



Solution: $\angle 1 + \angle 2 = \angle AOP$

$\angle 2 + \angle 2 = \angle AOP$ ...(i)

$(\because \angle 1 = \angle 2)$

Similarly, $2\angle 4 = \angle BOP$ ...(ii)

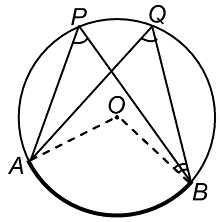
Adding (i) and (ii), we have

$2[\angle 2 + \angle 4] = \angle AOP + \angle BOP = 180°$

$\Rightarrow$ $2\angle ACB = 180°$

$\Rightarrow$ $\angle ACB = 90°$ [a right angle]

Theorem 9 : Angles in the same segment of a circle are equal.



Given : $\angle APB$ and $\angle AQB$ are two angles subtended by $\overset{\frown}{AB}$ in the same segment of the circle. *O* is the centre of the circle.

To prove : $\angle APB = \angle AQB$

Construction : Join *OA* and *OB*

Proof : $\angle AOB = 2\angle APB$ …(i)

$\angle AOB = 2\angle AQB$ …(ii)

From (i) and (ii)

$2\angle APB = 2\angle AQB$

Therefore, $\angle APB = \angle AQB$.

Theorem 10 : The sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Theorem 11 : (Converse of Theorem 10) If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.