1. QUADRATIC EQUATIONS

A quadratic equation is a second degree polynomial in x usually equal to zero where $a \neq 0$ and a, b, c are real numbers.

Therefore, $ax^2 + bx + c = 0, a \neq 0$ is called the standard form of a quadratic equation.

e.g. $2x^2 - 3x + 7 = 0, \quad 8x^2 + x - \sqrt{19} = 0$

If there is written only $2x^2 - 3x + 7$ 2 then it is not a quadratic equation it is called expression.

1.1 ZEROS OF QUADRATIC POLYNOMIAL

The values of x that satisfies then equation, are called zeros of quadratic polynomial $p(x)$

i.e. if $P(\alpha)$ = $a\alpha^2 + b\alpha + c = 0$ then α is called the zero of quadratic expression.

1.2 ROOTS OF QUADRATIC EQUATION

If α, β are zeros of polynomial $ax^2 + bx + c,$ then α, β are called roots (or solutions) of corresponding equation $ax^2 + bx + c = 0$ which implies that $p(\alpha) = p(\beta) = 0.$

i.e., $a\alpha^2 + b\alpha + c = 0$ and $a\beta^2 + b\beta + c = 0.$

In general the roots of a quadratic equation can be found in two ways.

(i) by factorizing the expression.

(ii) by using the standard formula.

2. SOLUTION OF A QUADRATIC EQUATION BY FACTORISATION

If a and b are numbers, then $ab = 0$ implies that either $a = 0$ or $b = 0$

If the quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$, a, b, c ER is given then if it has roots $x = \alpha$ and $a = \beta$ then if can be written as $a(x - \alpha)(x - \beta) = 0$

$\Rightarrow \quad a[x^2 - (a + \beta)x + \alpha\beta] = 0$

$\Rightarrow \quad ax^2 - a(\alpha + \beta)x + a\alpha\beta = 0$

Comparing we get $\frac{9}{a} = \frac{b}{-a(\alpha + \beta)} = \frac{c}{a\alpha\beta}$

Comparing (i) and (ii) we get $\alpha + \beta = -\frac{b}{a}$

Comparing (i) and (iii) we get $\alpha\beta = \frac{c}{a}$

Hence sum of roots = $-\frac{b}{a}$ and product of roots = $\frac{c}{a}$.

Example : $x^2 - 5x + 6 = 0$

Comparing with $ax^2 + bx + c = 0$ we get

$a = 1 \quad b = -5 \quad c = 6$

Then $\alpha + \beta$ = – $\frac{(-5)}{1}$ and $\alpha\beta = \frac{6}{1}$

$\alpha + \beta = 5$ and $\alpha + \beta = 6$

Then it is only possible when

$\alpha = 2$ and $\beta = 3$ or $\alpha = 3$ and $\beta = 2$

Hence 2 and 3 are roots of given equation $x = \frac{1}{a^{21}} = \frac{1}{b^2}$

3. SOLUTION OF A QUADRATIC EQUATION BY COMPLETING THE SQUARE

Following steps are involved in solving a quadratic equation by quadratic formula

- Consider the equation $ax^2 + bx + c = 0$, where a ≠ 0
- Dividing throughout by '*a*', we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

- Add and subtract $\left(\frac{1}{2}\text{ coefficient of } x\right)^2$ for making perfect square, we get

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$,

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a} = \frac{b^2 - 4ac}{4a^2}$$

- If $b^2 - 4ac \geq 0$ taking square root of both sides, we obtain

$$x + \frac{b}{2a} = \frac{\pm\sqrt{b^2 - 4ac}}{2a}$$

Therefore $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

The Quadratic Formula: Quadratic equation, $ax^2 + bx + c = 0,$ where a, b, c are real number and $a \neq 0$, has the roots as

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$ (Also known as Sri Dharacharya Rule)

4. NATURE OF ROOTS

In previous section, we have studied that the roots of the equation $ax^2 + bx + c = 0$ are given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

The nature of roots are only determined by the value of $b^2 = 4ac$, it is called the "Discriminant' of the quadratic equation.

A quadratic equation $ax^2 + bx + c = 0$ has

· **Two distinct real roots if $b^2 - 4ac > 0$.**

If $b^2 - 4ac > 0,$ we get two distinct real roots $-\frac{b}{2a} + \frac{\sqrt{b^2 - 4ac}}{2a}$ and $-\frac{b}{2a} - \frac{\sqrt{b^2 - 4ac}}{2a}$

Condition - (i)	Condition - (ii)
If $b^2 - 4ac$ is a perfect square then the roots are real and rational if a, b, c E rational.	If $b^2 - 4ac$ is not a perfect square then the roots are real and irrational and the occur in conjugate pairs

· **Two equal roots, if $b^2 - 4ac = 0$.**

If $b^2 - 4ac = 0,$ then $x = -\frac{b \pm 0}{2a}$; i.e. $x = -\frac{b}{2a}$

So, the roots are both $-\frac{b}{2a}$

· **No real roots, if $b^2 - 4ac < 0$**

Hence If $b^2 - 4ac < 0$, then the roots are complex conjugates i.e., if one root is $2 + \sqrt{3}i$ then other it $2 - \sqrt{3}i$

If $b^2 - 4ac < 0,$ then there is no real number whose square is $b^2 - 4ac.$

Note : (i) whenever the roots of quadratic equation are irrational, $(a, b, c\, E \text{ Rational})$, are of the form $a+\sqrt{b}$ and $a-\sqrt{b}$, i.e., whenever $a+\sqrt{b}$ is one roots of quadratic equation, then $a-\sqrt{b}$ is the other root of quadratic equation and vice versa.

5. CONSTRUCTING A QUADRATIC EQUATION

We can build a quadratic equation in the following cases :

(i) When the roots of the quadratic equation are given.

(iii) When the sum of the roots and the product of roots of the quadratic equation are given.

Case (i) : If the roots of the quadratic equation are α and β, then its equation can be written as $(x-\alpha)(x-\beta)=0$ i.e., $x^2 - x(\alpha+\beta)+\alpha\beta=0$.

Case (ii) : If p is the sum of the roots of the quadratic equation and q is their product, then the equation can be written as $x^2 - px + q = 0$.

Constructing a new quadratic equation by changing the roots of a given quadratic equation.

(i) A quadratic equation whose roots are $\frac{1}{\alpha}$ and $\frac{1}{\beta}$. i.e., the roots are reciprocal to the roots of the given quadratic equation can be obtained by substituting $\frac{1}{x}$ for x in the given equation.

(ii) A quadratic equation whose roots are $(\alpha+k)$ and $(\beta+k)$ can be obtained by substituting $x-k$ for x in the given n equation.

(iii) A quadratic equation whose roots are $(\alpha-k)$ and $(\beta-k)$ can be obtained by substituting $x+k$ for x in the given n equation.

(iv) A quadratic equation whose roots are $k\alpha$ and $k\beta$ can be obtained by substituting $\frac{x}{k}$ for x in the given equation.

(v) A quadratic equation whose roots are $\frac{\alpha}{k}$ and $\frac{\beta}{k}$ can be obtained by substring kx for x in the given equation.

6. MAXIMUM OR MINIMUM VALUE OF QUADRATIC EXPRESSION

The quadratic expression ax^2+bx+c a takes different values as x takes different values.

For all the values of x, as x varies from $-\infty$ to $+\infty$ (i.e., when x is real) the quadratic expression $ax^2 + bx + c$.

(i) has a minimum value if $a > 0$ (i.e., a is positive)

The minimum value of the quadratic expression is $\frac{4ac - b^2}{4a}$ and it occurs at $x = -\frac{b}{2a}$.

(ii) has a maximum value if $a < 0$. The maximum value of expression is $\frac{4ac - b^2}{4a}$ and it occurs at $x = -\frac{b}{2a}$